Some Basics of Machine Learning

Qijia He, PhD student
Time: 10:00am – 11:00am, June 25, 2026

Slide Credits

1. *Introduction to Machine Learning*, 10-401, by Maria-Florina Balcan, Carnegie Mellon University: https://www.cs.cmu.edu/~ninamf/courses/401sp18/lectures.shtml

2. *Machine Learning*, CS4/5780, by Kilian Weinberger, Cornell University: https://www.cs.cornell.edu/courses/cs4780/2017sp/

3. *Intro to Machine Learning*, CS4/CS5780, by Wen Sun, Cornell University: https://www.cs.cornell.edu/courses/cs4780/2023fa/

4. Deep Learning for Computer Vision, CS231n, by Fei-Fei Li et. al., Standford University: https://cs231n.stanford.edu

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- Part I: Linear Classifier
- Part II: Linear Regression
- Part III: Gradient Descent (and Beyond)
- Part IV: Neural Networks

Part I: Linear Classifier

Classification

Name	Math	Grade
Alice	90	A
Bob	88	A
Eric	82	A
Lily	77	B
Dick	62	B
Lucy	**76**	**?**

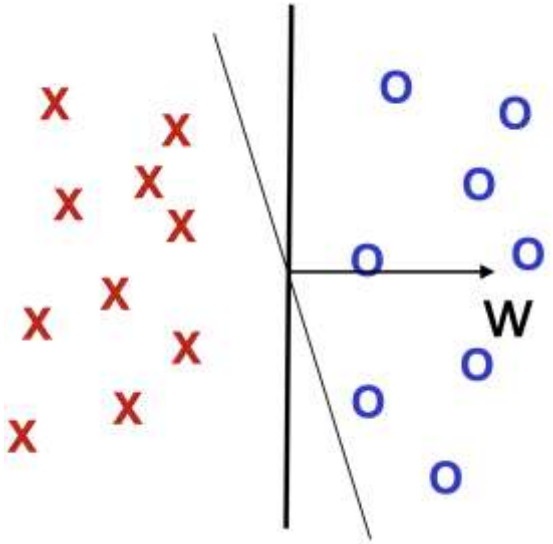
Name	Math	Biology	Art	History	Grade	Total Score
Alice	90	100	65	88	A	343
Bob	88	67	71	56	B	282
Eric	82	87	98	89	A	356
Lily	77	90	79	82	A	328
Dick	62	55	78	61	B	266
Lucy	**76**	**98**	**85**	**92**	**?**	341

KNN (K nearest neighbor)
Separator based Algorithms
Linear Classification

High level idea of Linear Classification:
Treat each subject score as a feature, look at the expert's grades, and learn a weighted combination of those features so that your prediction best matches the expert's data.

Linear Separators

- Feature space $X \in \mathbb{R}^d$.
- Hypothesis class of linear decision boundary in $\mathbb{R}^d$.
  - $h(x) = \boldsymbol{W}^T x + w_0$, where $\boldsymbol{W} = (w_1, \dots, w_d) \in \mathbb{R}^d$
  - If $h(x) \geq 0$, then label x as +, otherwise label it as –



How could we efficiently "learn" the weight ($\boldsymbol{W}$) of those features?

- When observing a new data with label, if the ground truth label is positive but we give a wrong prediction (h(x)<0), we want the boundary to be higher (let $h(x) + +$, such that next time it can have a higher probability to be predicted as positive), same for negative ($h(x) - -$)
- How to increase/decrease $h(x)$? One way is to adjust the weight with $\boldsymbol{W_{t+1}} \leftarrow \boldsymbol{W_t} \pm x$

  Positive samples: $\boldsymbol{W}_{t+1}^T x \leftarrow \boldsymbol{W}_t^T x + x^T x = \boldsymbol{W}_t^T x + ||x||^2$

  Negative samples: $\boldsymbol{W}_{t+1}^T x \leftarrow \boldsymbol{W}_t^T x - x^T x = \boldsymbol{W}_t^T x - ||x||^2$

Linear Separators: Perceptron Algorithm

- Set $t = 1$, start with the all zero vector $\boldsymbol{W_1} = (0, \ldots, 0)$.
- Given example x, predict positive iff $\boldsymbol{W}_t^T x \geq 0$.
- On a mistake, update as follows:
  - Mistake on positive, then update $\boldsymbol{W}_{t+1} \leftarrow \boldsymbol{W}_t + x$
  - Mistake on negative, then update $\boldsymbol{W}_{t+1} \leftarrow \boldsymbol{W}_t - x$

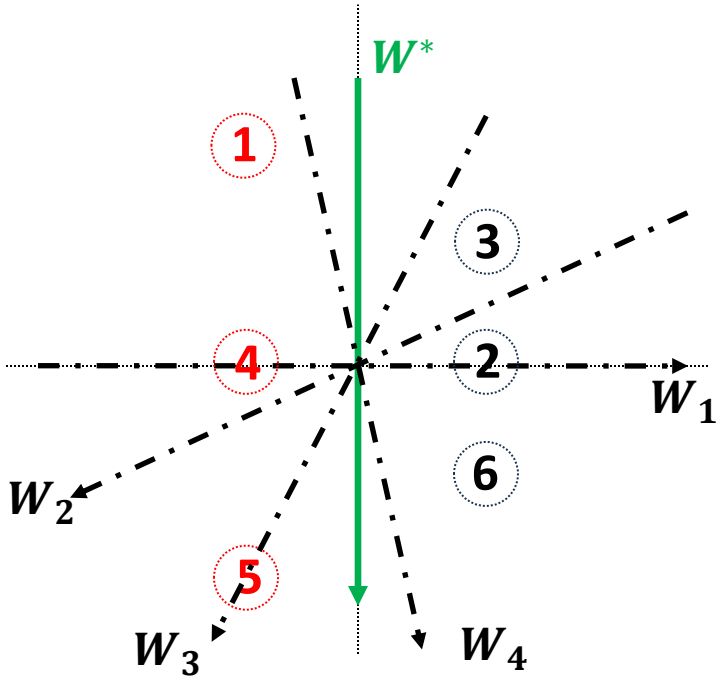
Natural ***greedy*** procedure:

- If true label of x is +1 and $\boldsymbol{W}_t$ incorrect on x, we have $\boldsymbol{W}_t^T x < 0$.
- By Perceptron Algorithm, we have $\boldsymbol{W}_{t+1}^T x \leftarrow \boldsymbol{W}_t^T x + x^T x = \boldsymbol{W}_t^T x + ||x||^2$.
- Then, there will be more chance that $\boldsymbol{W}_{t+1}$ classifies x correctly.
- Similar for mistakes on negative examples.

Perceptron Algorithm: Practice

Example: $W^* = (1, 0)$

1.	$(-1,2)$	-	$W_1 = (0,0)$	✗
2.	$(1,0)$	+	$W_2 = (1,-2)$	✓
3.	$(1,1)$	+	$W_2 = (1,-2)$	✗
4.	$(-1,0)$	-	$W_3 = (2,-1)$	✓
5.	$(-1,-2)$	-	$W_3 = (2,-1)$	✗
6.	$(1,-1)$	+	$W_4 = (3,1)$	✓



Algorithm:

- Set $t = 1$, start with the all zero vector $W_1 = (0, \dots, 0)$.
- Given example x, predict positive iff $W_t^T x \geq 0$.
- On a mistake, update as follows:
  - Mistake on positive, then update $W_{t+1} \leftarrow W_t + x$
  - Mistake on negative, then update $W_{t+1} \leftarrow W_t - x$

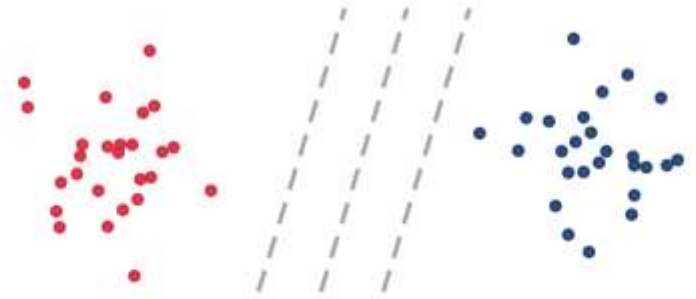
$W_1 = (0,0)$

$W_2 = W_1 - (-1,2) = (1,-2)$

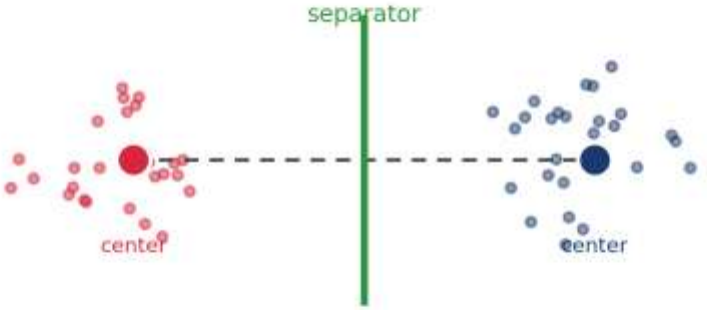
$W_3 = W_2 + (1,1) = (2,-1)$

$W_4 = W_3 - (-1,-2) = (3,1)$

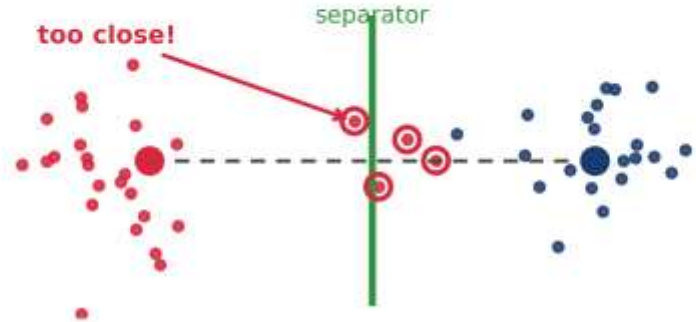
What is a good separator?



There's a lot of ways to separate the two clusters…



Keeping the center of each cluster as far from the separator as possible?



Not a good idea…

We introduce the concept of margin

What Is the Margin?

- The **margin** is the distance from the line to the closest data point — the width of that buffer.
- A bigger margin means more breathing room, and a more confident boundary.

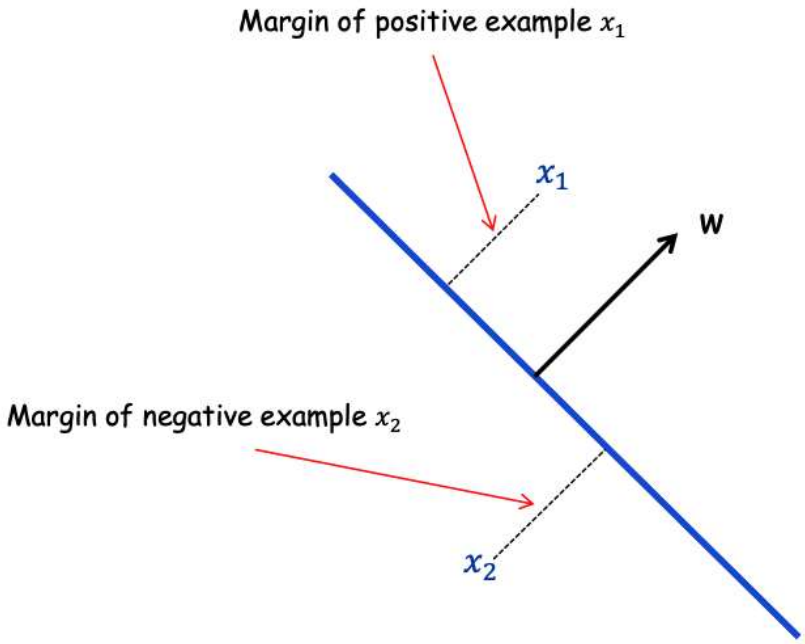
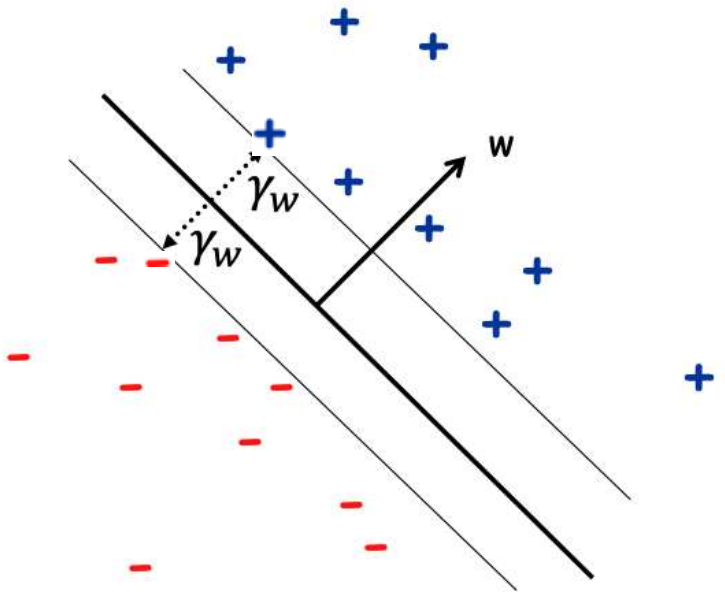


Figure from Balcan (2018)

Support Vector Machines (SVM)

- Many different lines can separate the two groups — but they are not equally good.
- A **Support Vector Machine (SVM)** picks the line with the **largest margin**: the one sitting in the widest possible "street" between the two groups.
- The few points that just touch the edges of the street are the **support vectors** — they alone decide where the line goes.

- Input: $S = \{(x_1, y_1), \dots, (x_m, y_m)\}$;
- Find: some $\boldsymbol{W}$ and maximum γ where:
  - $\left|\left|\boldsymbol{W}\right|\right| = 1$
  - For all $i \in \{1, \dots, m\}, y_i \boldsymbol{W}^T x_i \geq \gamma$
- Output: maximum margin separator over S.



Part II: Linear Regression

From Discrete to Continuous Labels

Classification:

Sports
Science
News

X = Document

Y = Topic

Regression:

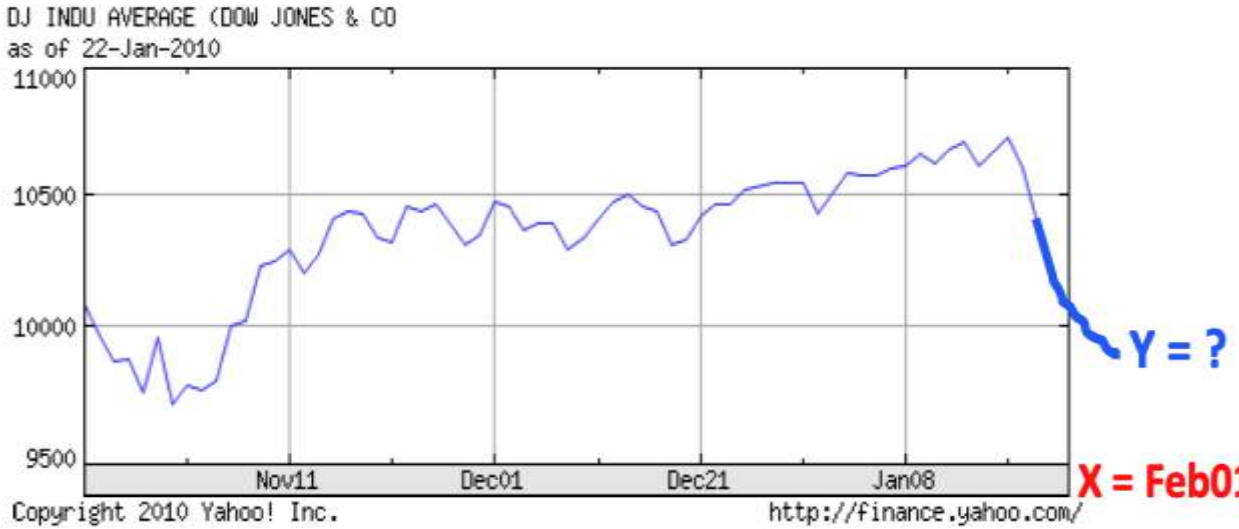


Figure from Balcan (2018)

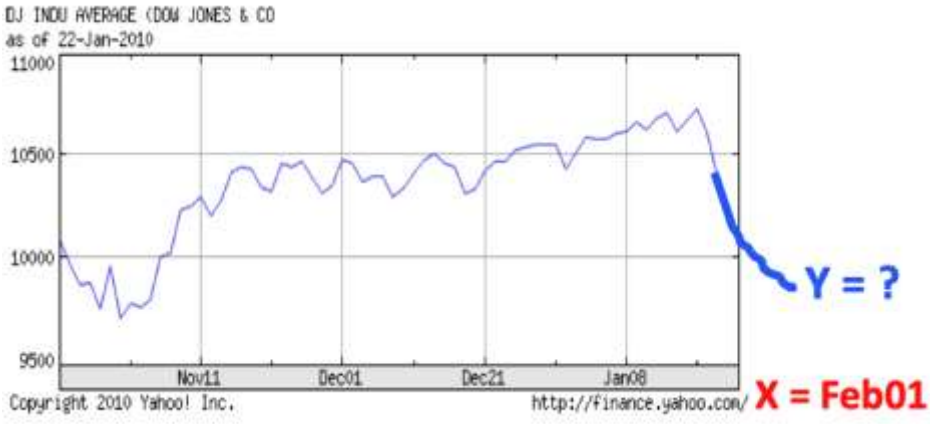
Supervised Learning

Goal: Construct a predictor $f: X \rightarrow Y$ to minimize a risk (error measure) err(f) .

X = Document

Sports
Science
News

Y = Topic



Classification:

$$\text{err}(f) = P(f(X) \neq Y)$$

Probability of error

Regression:

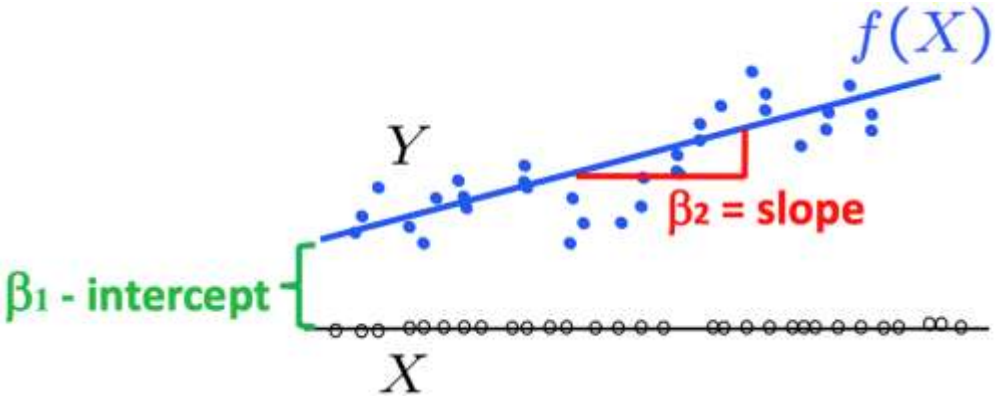
$$\text{err}(f) = E[(f(X) - Y)^2)]$$

Mean Squared Error

Figure from Balcan (2018)

Linear Regression

- Unit-variate case: $f(X) = \beta_1 + \beta_2 X$.



- Multi-variate case:

$$f(X) = f\left(X^{(1)}, \ldots, X^{(p)}\right) = \beta_1 X^{(1)} + \beta_2 X^{(2)} + \ldots + \beta_p X^{(p)}$$
$$= X\beta, \text{ where } X = \left[X^{(1)} \ldots X^{(p)}\right], \beta = [\beta_1 \ \ldots \beta_p]^T$$

- Least square estimator: $\hat{f} = \arg\min_{f \in F} \frac{1}{n} \sum_{i=1}^{n} (f(X_i) - Y_i)^2$, where F is the class of linear functions.

Figure from Balcan (2018)

Least Squares Estimator

- $\hat{f} = \arg\min_{f \in F} \frac{1}{n} \sum_{i=1}^{n} (f(X_i) - Y_i)^2$

- $\hat{\beta} = \arg\min_{\beta} \frac{1}{n} \sum_{i=1}^{n} (X_i\beta - Y_i)^2$

  $= \arg\min_{\beta} \frac{1}{n} (\mathbf{X}\beta - \mathbf{Y})^T (\mathbf{X}\beta - \mathbf{Y})$

$$\mathbf{X} = \begin{bmatrix} X_1 \\ \dots \\ X_n \end{bmatrix} = \begin{bmatrix} X_1^{(1)} & \dots & X_1^{(p)} \\ \vdots & \ddots & \vdots \\ X_n^{(1)} & \dots & X_n^{(p)} \end{bmatrix}, \quad \mathbf{Y} = \begin{bmatrix} Y_1 \\ \dots \\ Y_n \end{bmatrix}.$$

Least Squares Estimator

- $\hat{\beta} = \arg\min_{\beta} \frac{1}{n}(\mathbf{X}\beta - \mathbf{Y})^T(\mathbf{X}\beta - \mathbf{Y}) = \arg\min_{\beta} J(\beta)$
- $J(\beta) = (\mathbf{X}\beta - \mathbf{Y})^T(\mathbf{X}\beta - \mathbf{Y})$
- $\frac{\partial J(\beta)}{\partial \beta}|_{\hat{\beta}} = 0$

- $(\mathbf{X}^T\mathbf{X})\hat{\beta} = \mathbf{X}^T\mathbf{Y}$
- If $\mathbf{X}^T\mathbf{X}$ is invertible,

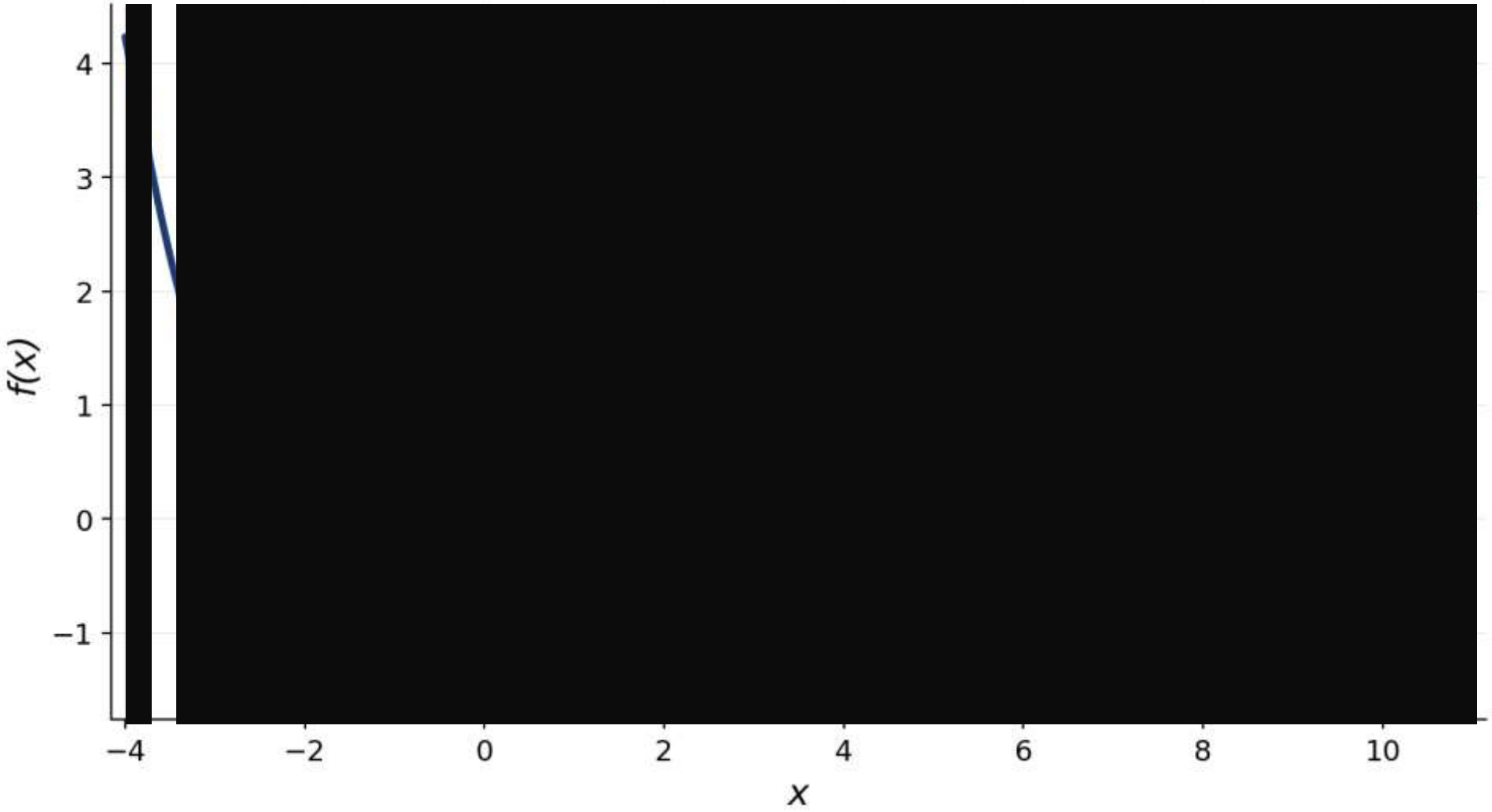
$$\hat{\beta} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y} \qquad \hat{f}(\boldsymbol{X}) = \mathbf{X}\hat{\beta}$$

It is lucky that linear regression has such a nice closed form solution. However, for most ML problems, we cannot express the result in a mathematical form

Most of time, The model is like a black box: We can calculate the predicted value given X, taking derivative, but can not find solution for $\frac{\partial J(\beta)}{\partial \beta}|_{\hat{\beta}} = 0$.

Part III: Gradient Descent (and Beyond)


X
Model
???
y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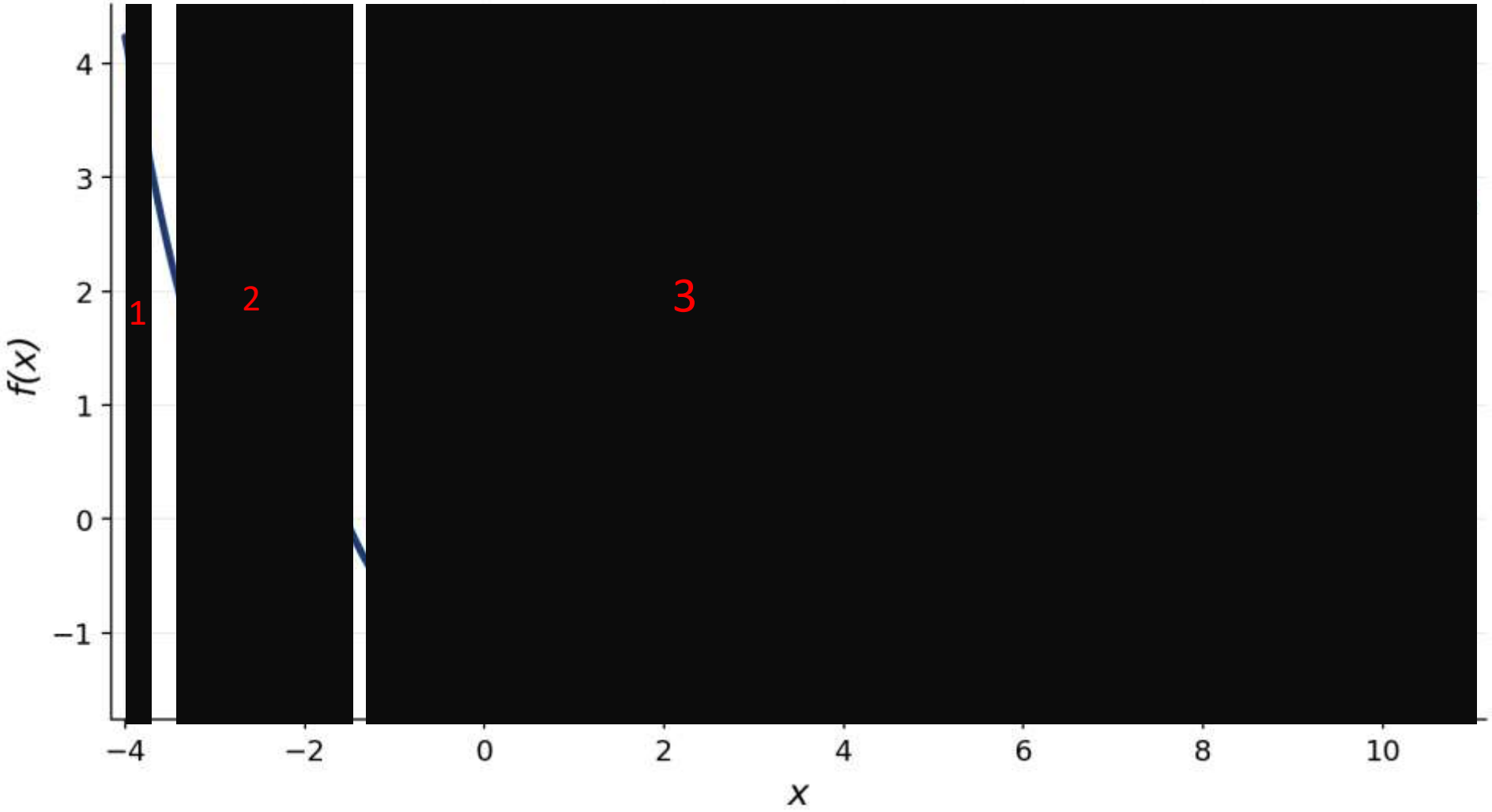
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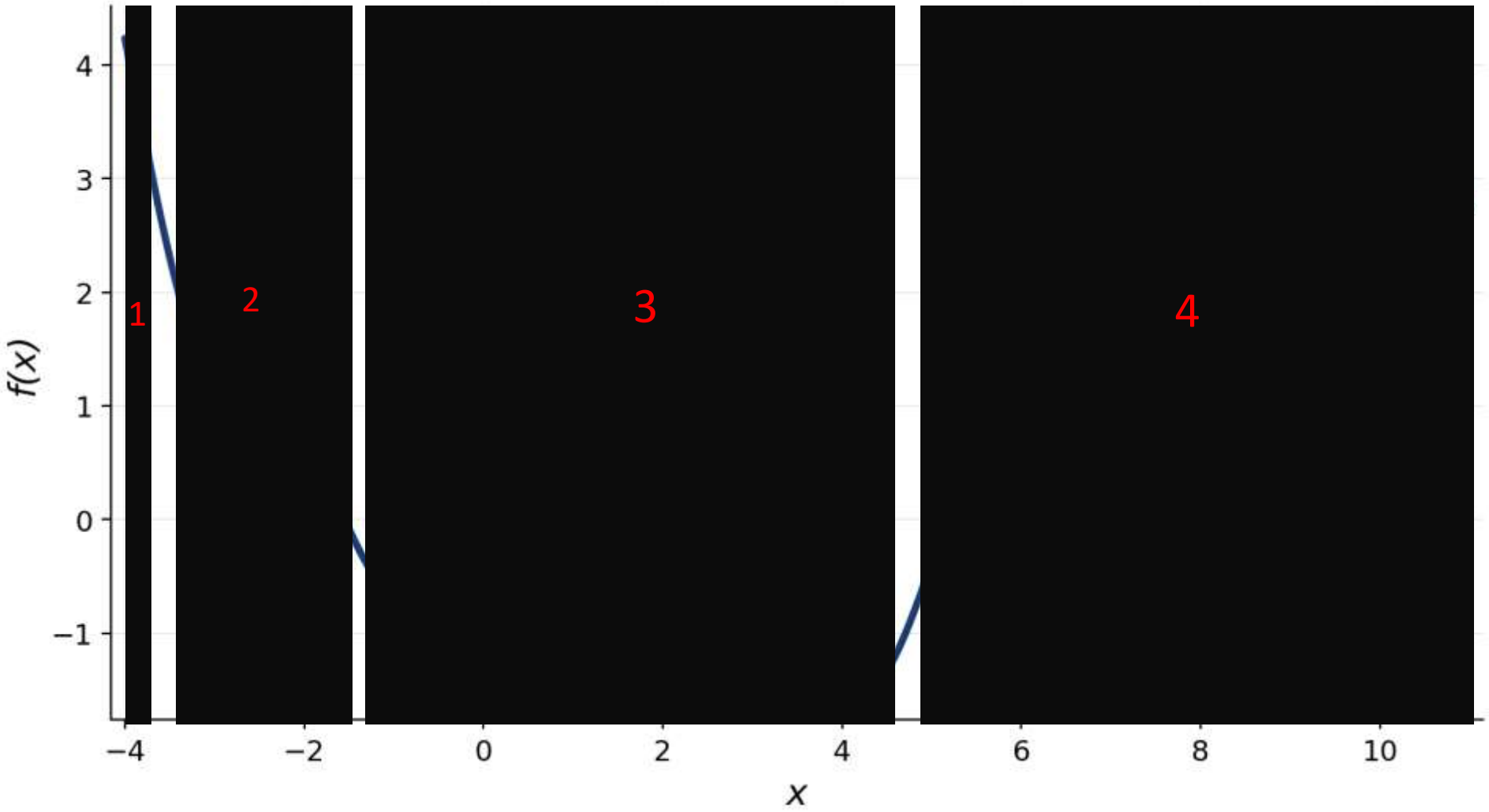
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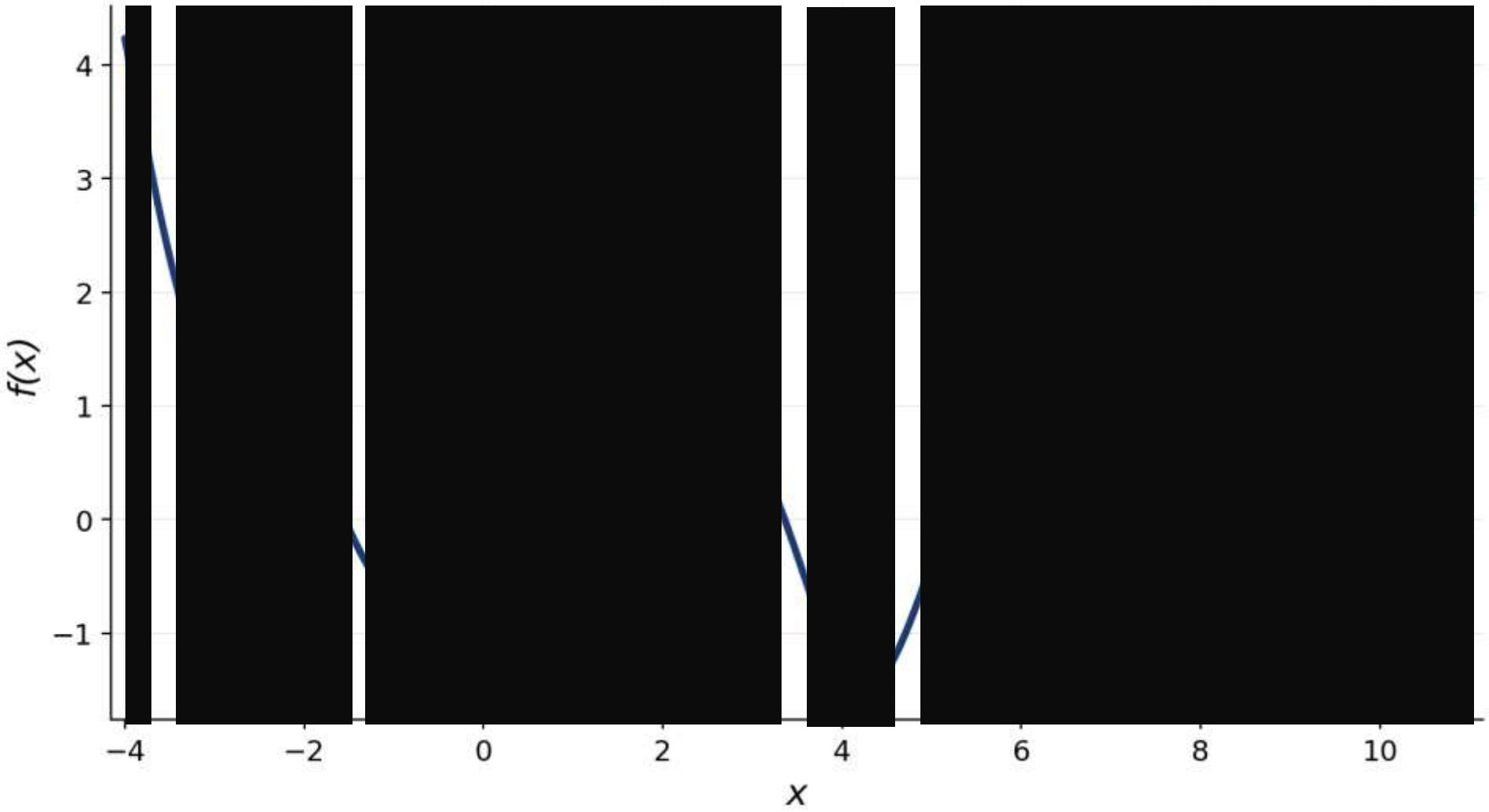
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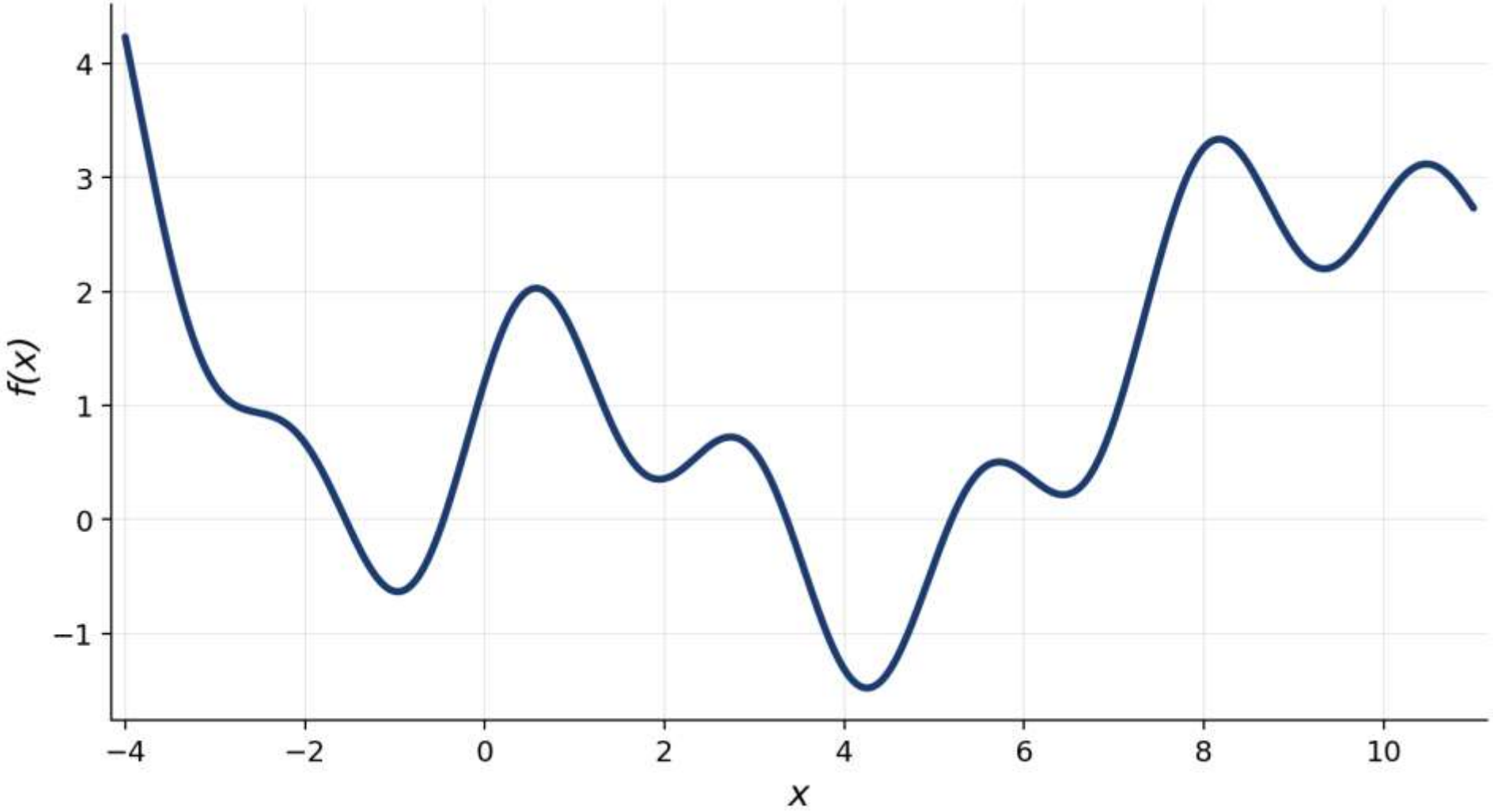
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0
−1
f(x)
−4
−2
0
2
4
6
8
10
x

Gradient Descent (GD)

- In GD, we only use the gradient (first order).
- We assume the function l around $\boldsymbol{W}$ is linear and behaves like $l(\boldsymbol{W}) + g(\boldsymbol{W}) \cdot s$.
- Our objective is to find a vector s that minimizes function l.
- In steepest descent we simply set

$$s = -\eta \cdot g(\boldsymbol{W})$$

for some small $\eta > 0$.

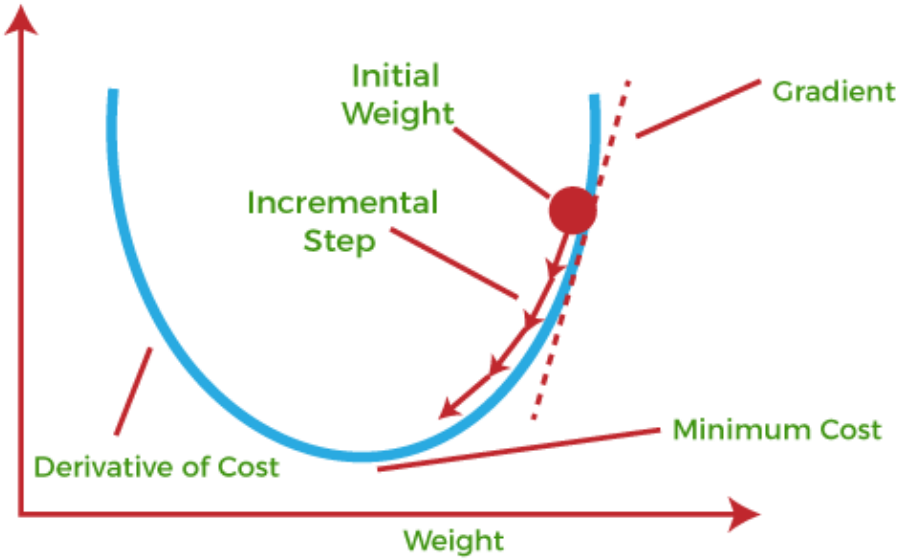
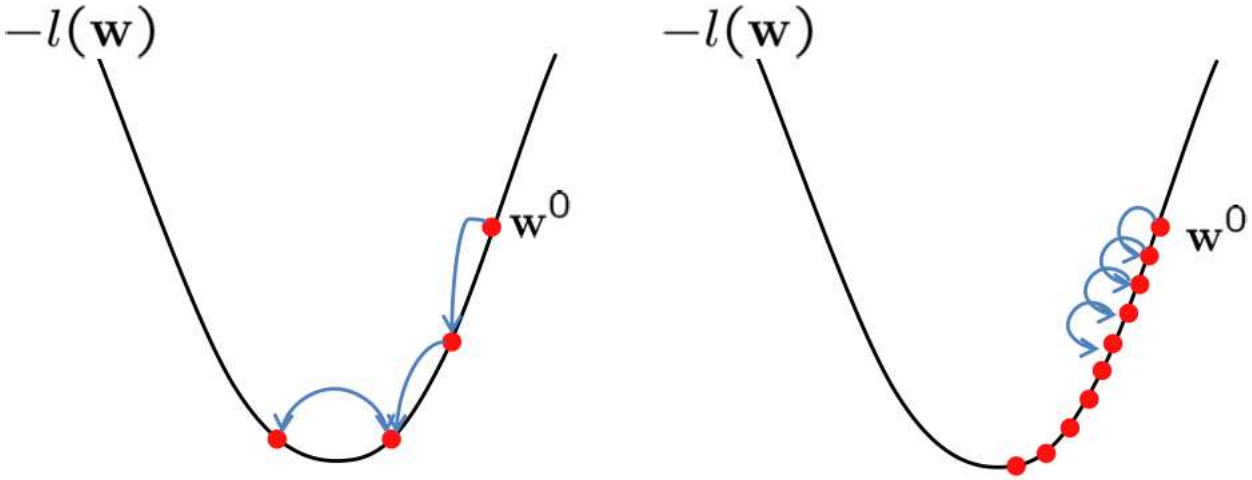


Figure from Kharkar (2023)

GD: Learning Rate

- Setting the learning rate $\eta > 0$ is a dark art.
  - Large $\eta \Longrightarrow$ Fast convergence but larger residual error $||\boldsymbol{W^{t+1}} - \boldsymbol{W^{t}}||_2$, with possible oscillations.
  - Small $\eta \Longrightarrow$ Slow convergence but small residual error.



- A safe (but sometimes slow) choice is to set $\eta = \frac{1}{t}$, which guarantees that it will eventually become small enough to converge.

Figure from Balcan (2018)

GD: In Practice

In ML, the loss we minimize typically has some special form, e.g.,

$$l(\boldsymbol{W}) = \frac{1}{n}\sum_{i=1}^{n} \ln(1 + \exp(-y_i(\boldsymbol{W}^T x_i)))$$

Average over n data points

To compute the gradient $\nabla l(\boldsymbol{W})$, we need to enumerate all n training data points, which can be very slow!

Stochastic GD (SGD) to Rescue

In ML, the loss we minimize typically has some special form, e.g.,

$$l(\boldsymbol{W}) = \frac{1}{n}\sum_{i=1}^{n} \ln(1 + \exp(-y_i(\boldsymbol{W}^T x_i)))$$

Average over n data points

Idea: Randomly sample a data point (x, y), use $\nabla l(x, y; \boldsymbol{W})$ to replace $\nabla l(\boldsymbol{W})$.

Stochastic GD (SGD)

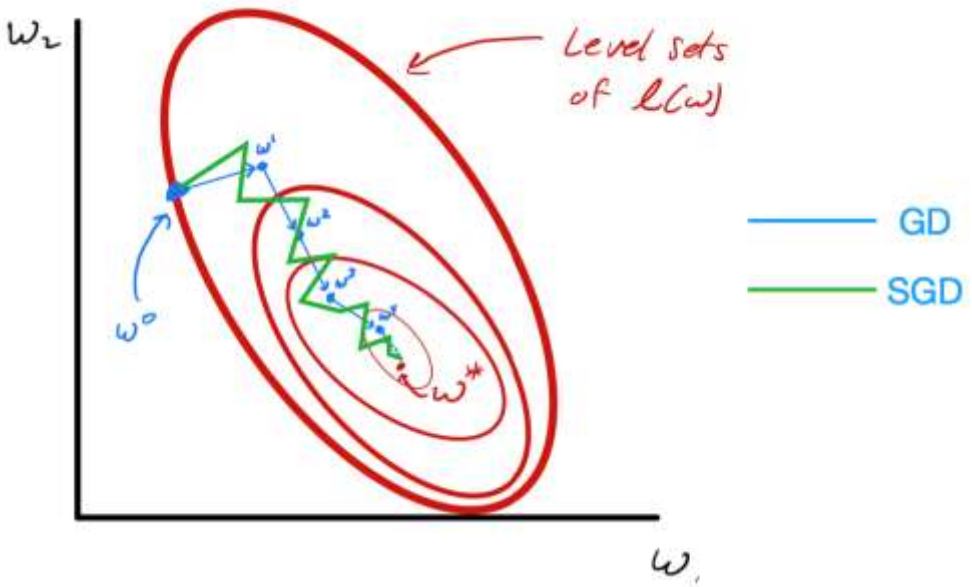
- Goal: minimize $l(\boldsymbol{W}) = \frac{1}{n}\sum_{i=1}^{n} l(x_i, y_i; \boldsymbol{W})$

- Initialize: $\boldsymbol{W}^0 \in \mathbb{R}^d$ randomly
- Iterate until convergence:
  1. Randomly sample a point (x_i, y_i) from the n data points
  2. Compute noisy gradient $\tilde{g}^t = \nabla l(x_i, y_i; \boldsymbol{W})|_{\boldsymbol{W}=\boldsymbol{W}^t}$
  3. Update (GD): $\boldsymbol{W^{t+1}} = \boldsymbol{W^t} - \eta \tilde{g}^t$

Why Can SGD Work?

Claim: the random noisy gradient is an unbiased estimate of the true gradient

$$\mathbb{E}[\nabla l(x_i, y_i; \boldsymbol{W})] = \frac{1}{n}\sum_{i=1}^{n} \nabla l(x_i, y_i; \boldsymbol{W}) = \nabla\left[\frac{1}{n}\sum_{i=1}^{n} l(x_i, y_i; \boldsymbol{W})\right] = \nabla l(\boldsymbol{W})$$



Q: Which one is faster?

A: SGD is slower in theory, but is often much faster in practice.

Figure from Sun (2022)

Revisiting Regression

$$\hat{\beta} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y}$$

- It might be computationally expensive if $\mathbf{X}$ is huge or $(\mathbf{X}^T\mathbf{X})^{-1}$ is not invertible.

$$\hat{\beta} = \arg\min_{\beta} \frac{1}{n}(\mathbf{X}\beta - \mathbf{Y})^T(\mathbf{X}\beta - \mathbf{Y}) = \arg\min_{\beta} J(\beta)$$

- Gradient Descent since $J(\beta)$ is convex
  - Initialize: β^0
  - Update: $\beta^{t+1} = \beta^t - \frac{\eta}{2}\mathbf{X}^T \frac{\partial J(\beta)}{\partial \beta}\Big|_t$
    $= \beta^t - \eta\mathbf{X}^T(\mathbf{X}\beta^t - \mathbf{Y})$
  - Stop: when some criterion met, e.g., fixed # iterations, or $\frac{\partial J(\beta)}{\partial \beta}\Big|_t < \varepsilon$.

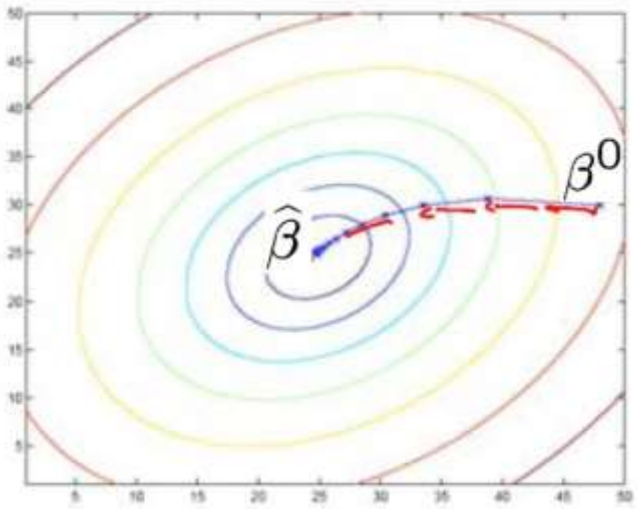


Figure from Balcan (2018)

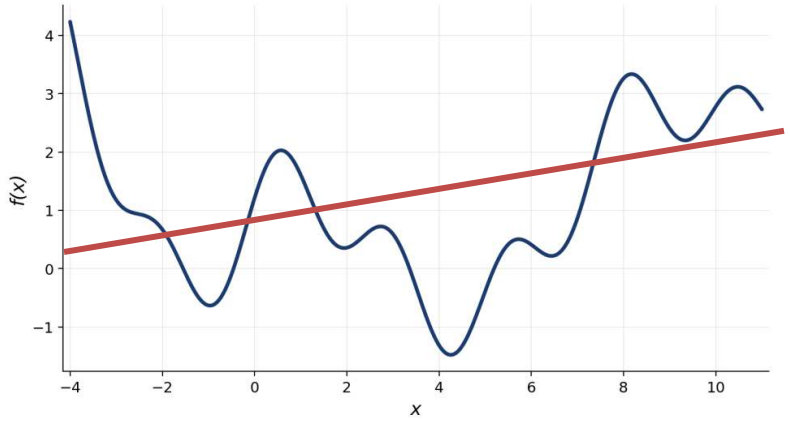
Part IV: Neural Networks

Neural Networks

Recall linear (regression) model:

$$\mathrm{Y} = \mathrm{Wx} + \mathrm{b}$$

Definitely not good for a non-linear model!



How about add some nonlinearity towards the linear feature?

$$\mathrm{Y} = \mathrm{f}(\mathrm{Wx} + \mathrm{b})$$

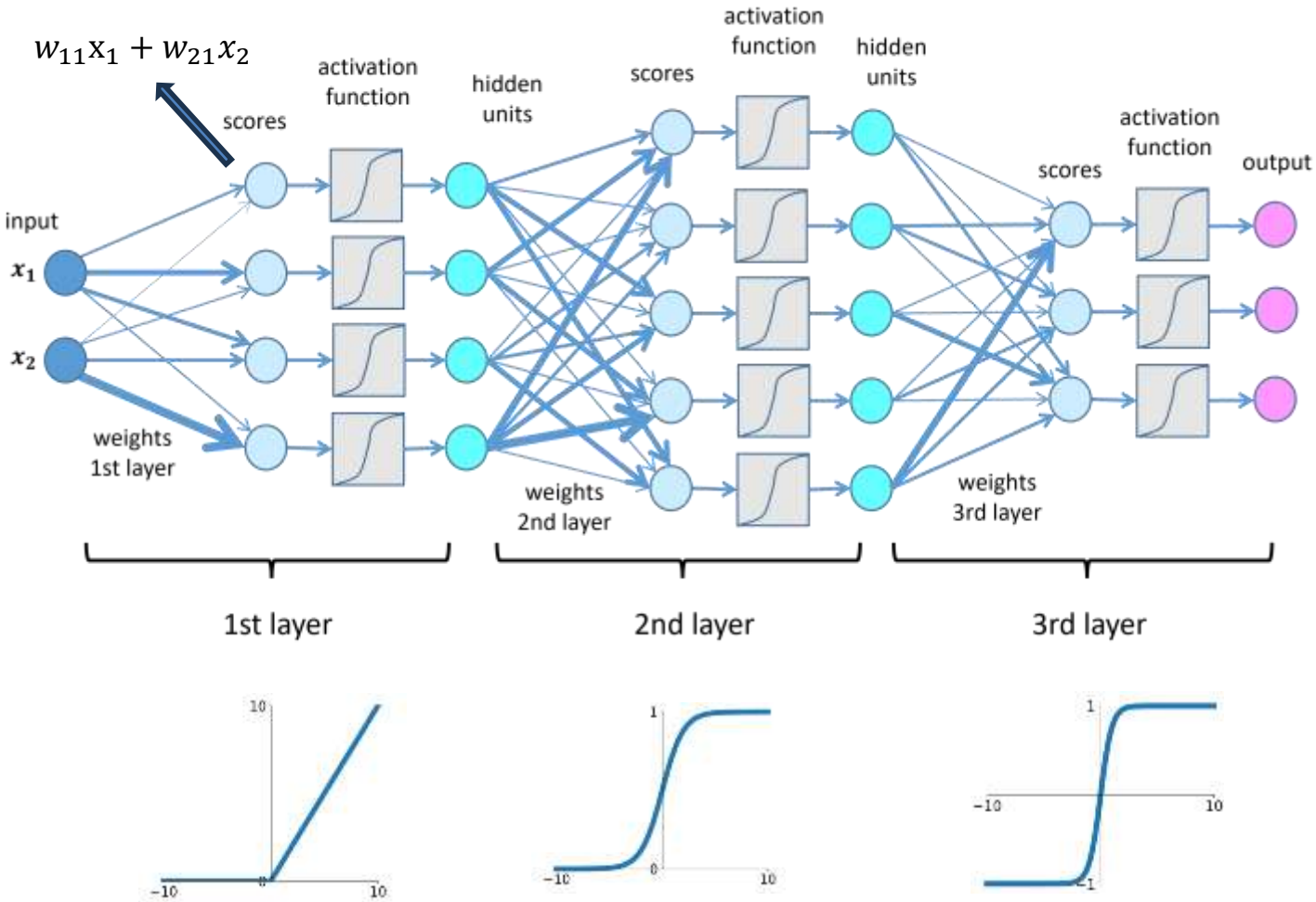
Where $\mathrm{f}(x)$ is a nonlinear function, i.e. e^x, $\frac{1}{1+\mathrm{e}^x}$

Perhaps one nonlinear function is not enough to capture the function space

Let's do more!

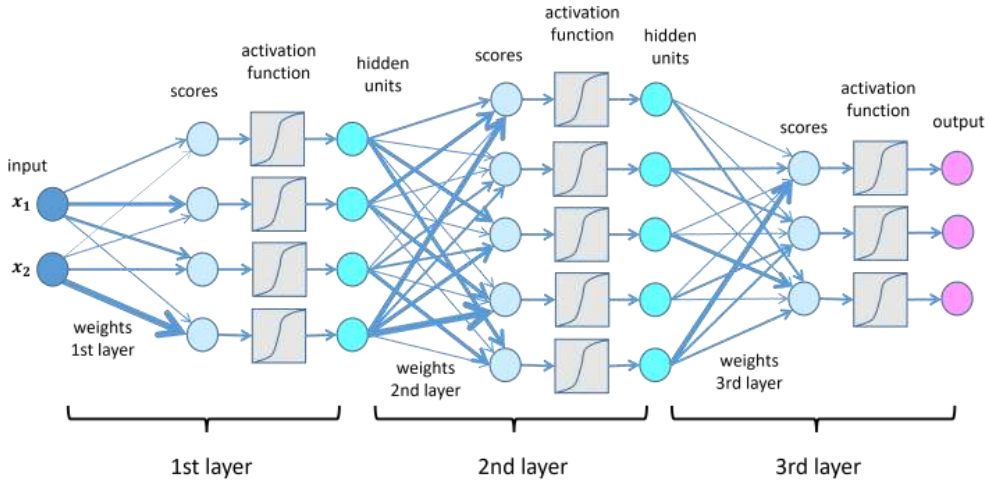
$$\mathrm{Y} = \mathrm{f}(\mathrm{Wx} + \mathrm{b})$$
$$\mathrm{Y} = \mathrm{f}(W\mathrm{f}(\mathrm{Wx} + \mathrm{b}) + \mathrm{b})$$
$$\mathrm{Y} = \mathrm{f}(W\mathrm{f}(W\mathrm{f}(\mathrm{Wx} + \mathrm{b}) + \mathrm{b}) + \mathrm{b})$$
$$\ldots$$

Neural Networks



- ReLU: $\sigma(x) = \max(0, x)$ Sigmoid: $\sigma(x) = \frac{1}{1+e^{-x}}$ tanh: $\sigma(x) = \tanh(x)$

Neural Networks



For each training steps:

1. We sample a batch of data (x1, x2)
2. Let the sampled data go through the Neural Networks, and get predictions (y1 y2)
3. We calculate the loss (e.g. MSE: $E[(f(X) - Y)^2)]$), and compute the gradients
4. Update the weights using Stochastic Gradient Descent

Language Task: Autoregressive Model

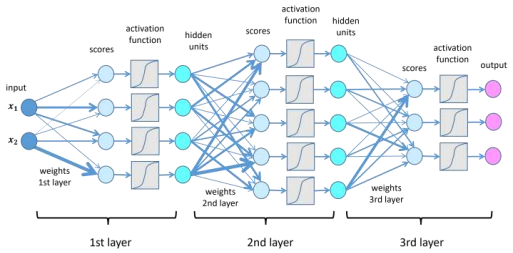
prefix

autoregressive language model (Encode and decode)

next-token distribution

Sampled token

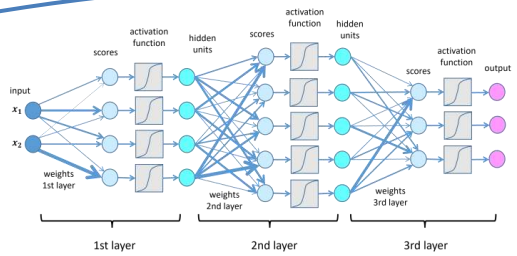
How are you



P(I'm|How are you)=0.6
P(good|How are you)=0.3
…

I'm

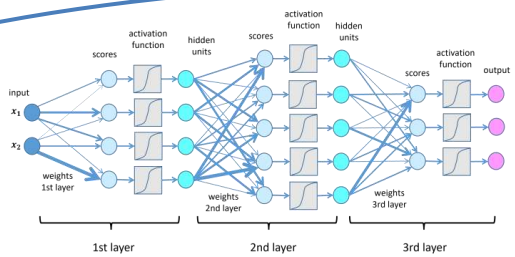
How are you. I'm



P(good|How are you. I'm)=0.5
P(fine|How are you. I'm)=0.45
…

fine

How are you I'm fine

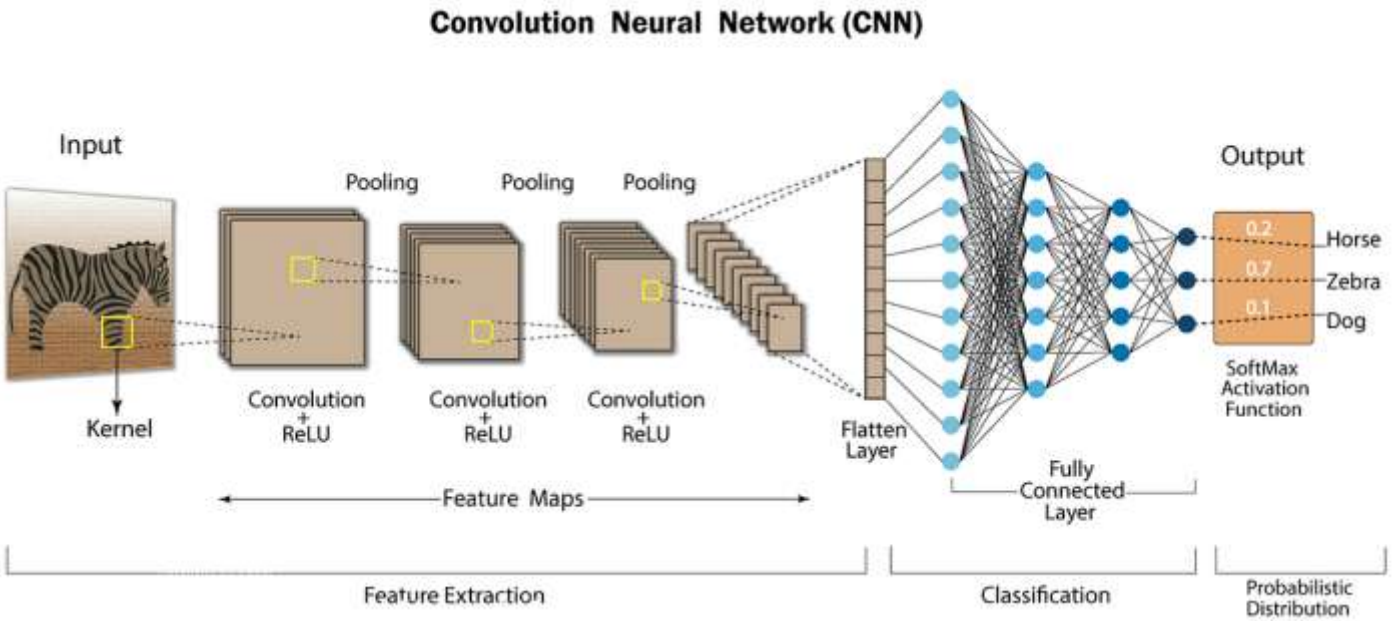


P(<EOS>|How are you. I'm fine)=0.8
P(Thank|How are you. I'm fine)=0.12
…

<EOS>

Final response: I'm fine ← External feedback: score=8

Vision task: Convolutional Neural Networks



Two new mechanisms:

1. Convolutional mechanism to capture the spatial information
2. Use Pooling to do dimensional reduction and extract key information

A Case of CNNs

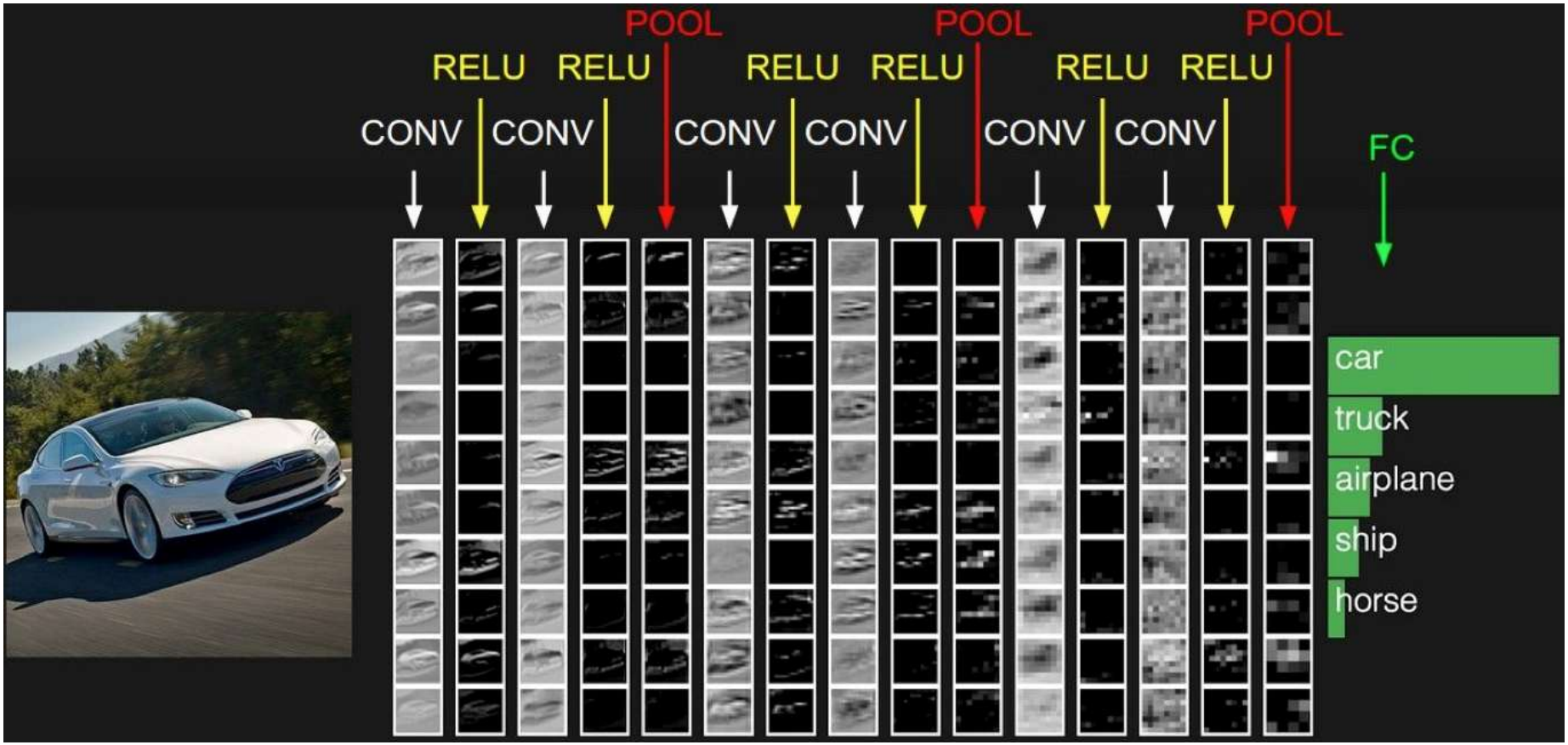


Figure from *Feifei Li & Andrej Karpathy (2016)*


thank you
danke
謝謝
ngiyabonga
teşekkür ederim
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bedankt
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dziękuję
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धन्यवाद

Illustration: Two-Dimensional Case

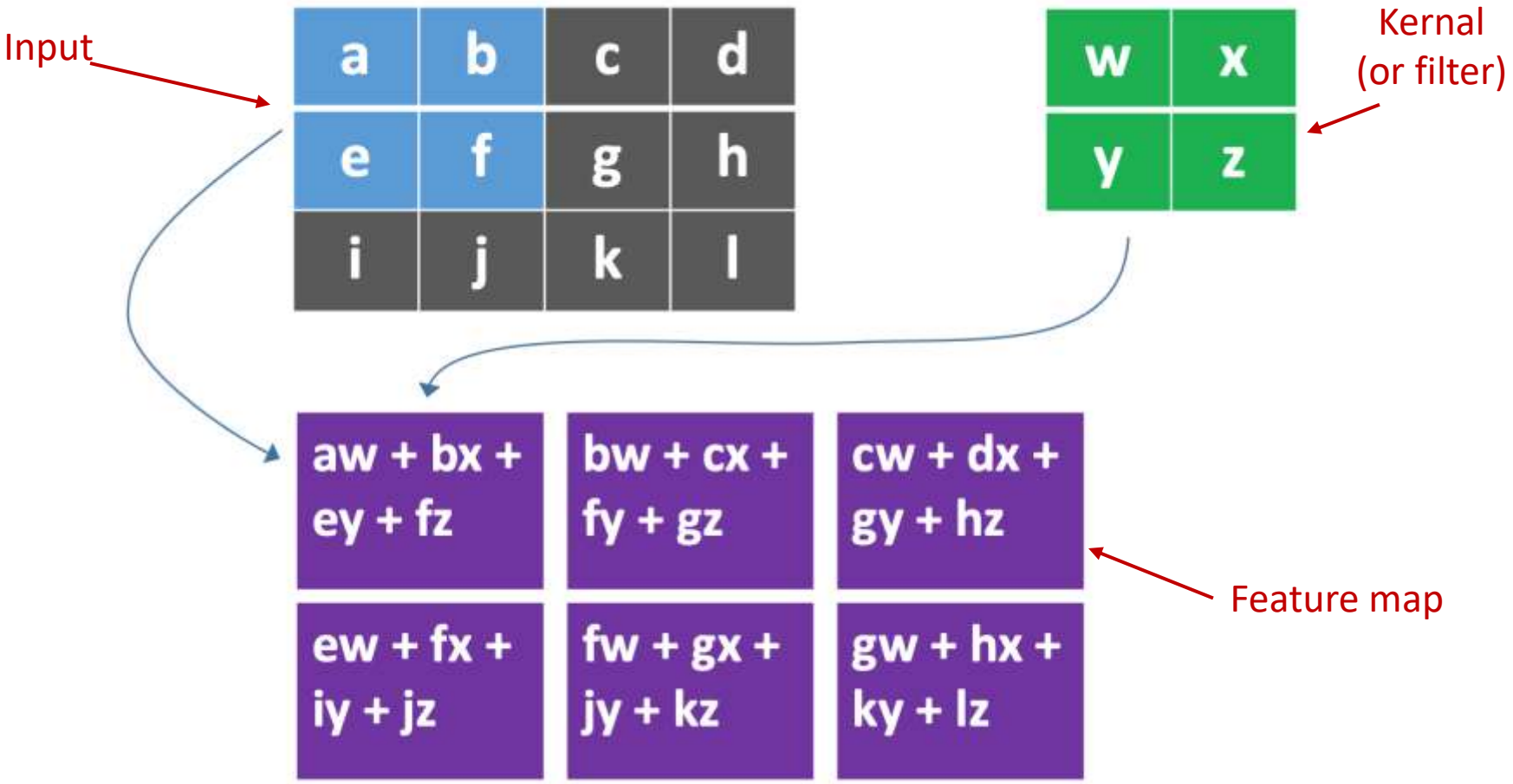
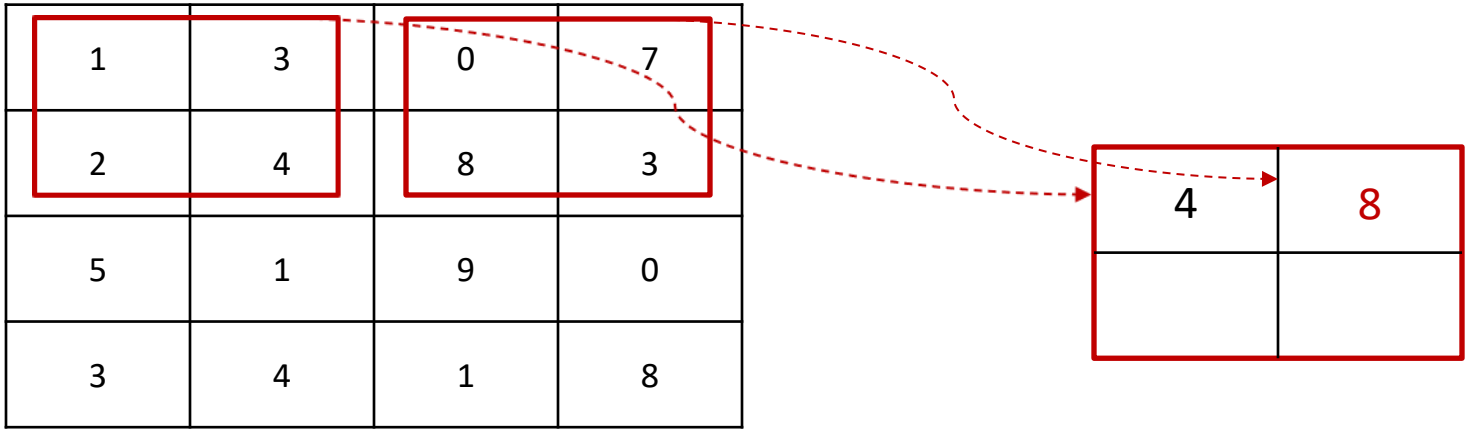


Figure from Balcan (2018)

Other Layers: Pooling Layer

- We use a pooling layer to downsize the inputs.
- For example, max pooling (2x2 filter and stride 2)

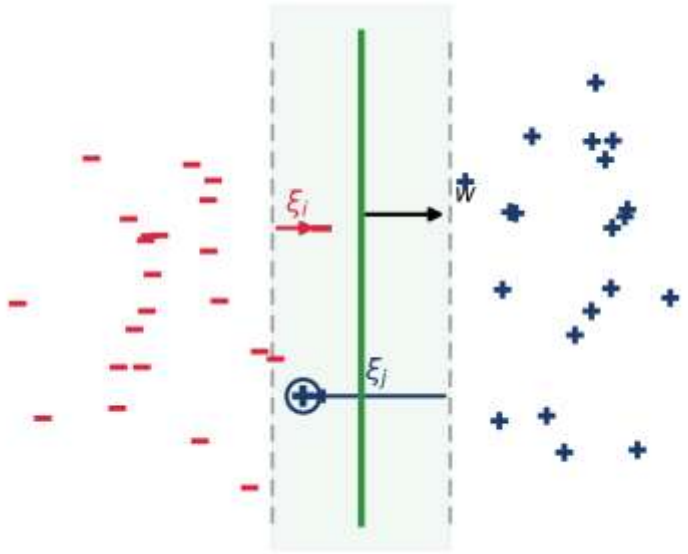


Cases where margin fails

We can't separate the two classes perfectly — so we allow a few mistakes, at a cost.

Soft margin: relax the constraint, penalize the slack

- Input: $S = \{(x_1, y_1), \ldots, (x_m, y_m)\}$;
- Find: some $\boldsymbol{W}$ and maximum $\gamma - C \sum_{i=1}^{m} \xi_i$ where:
  - $\left|\left|\boldsymbol{W}\right|\right| = 1$
  - For all $i \in \{1, \ldots, m\}, y_i \boldsymbol{W}^T x_i \geq \gamma - \xi_i$
- Output: maximum margin separator over S.



- ξ measures how far a point breaks the margin ($\xi > 1 \Rightarrow$ misclassified).
- Penalty $C \sum_{i=1}^{m} \xi_i$ trades off: large C $\Rightarrow$ fewer mistakes, narrow margin; small C $\Rightarrow$ wider margin.